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Solving a Partial Equilibrium Model in a CGE Framework: The Case of a Behavioural Microsimulation Model
APPENDIX

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A A SIMPLE APPROACH TO ITERATIVE SIMULATIONS FOR GEMPACK USERS

GEMPACK users can write the CGE model in percentage changes and the sub-model in levels in a single TABLO code. The CGE model is defined in the normal way using "VARIABLE" and "EQUATION" statements, while the BMS model is defined using "COEFFICIENT" and "FORMULA" statements in a separate section.

The CGE model is run first to produce a solution file. Those variables that are used as input into the BMS model, such as goods and factor prices, are written to a Header Array (HAR) file using the SLTOHT command. The sub-model then reads in the CGE shocks from this HAR file and writes its results to another HAR file to be used as exogenous changes in the CGE model in the next iteration. A batch file can be used to automate the procedure so that, in each simulation, the two models can run in sequence and also produces an input file for the next iteration.

This approach allows the BMS model to be written in levels within the same code as the CGE model. Therefore, more complex functional forms, such as conditional or discrete functions, and unconventional behavioural assumptions, such as switching between different options, can be

implemented alongside a linearised CGE equation system, which can save significant computation time.

B THE DATABASE OF AN INTEGRATED MODEL

The model is built on the basis of Australian input-output table 2012-13 (Australian Bureau of Statistics, 2015) and the 2009-10 Household Expenditure Survey (HES) and Housing and Income Surveys (HIS) (Australian Bureau of Statistics, 2012). The input-output table includes 104 industries and two factors of production, labour and capital. An aggregated version of this table is shown in Table B.1.

Table B.1: Aggregated input-output table in basic prices (AUD million).

	1 ind	2 hou	3 gov	4 inv	5 stk	6 exp	7 mgn	Total
1 dom	1,159,672	566,012	265,156	327,917	-2,034	270,578	272,793	2,860,094
2 imp	167,734	79,915	2,878	63,035	3,387			316,949
3 mgn	94,733	127,542	2,672	20,227	-87	27,706		272,793
4 GST	3,624	37,415	0	8,235	0	1,090		50,364
5 tax	17,831	25,502	0	13,609	652	0		57,594
6 sub	-6,636	-5,463	0	-1,154	-14	-616		-13,883
7 fac	1,371,108							1,371,108
8 ptx	52,028							52,028
Total	2,860,094	830,923	270,706	431,869	1,904	298,758	272,793	4,967,047

Notes: Basic values of imports (316,949) include import tariffs (3,233).
Source: Compiled from *Australian National Accounts: Input-Output Tables, 2012-13* (ABS 5209.0.55.001).

Industries (aggregated in the first column) use domestically produced and imported intermediate goods in production. Domestically produced goods (the first row) are used as intermediate inputs into current production by industries and other users: a representative household as consumer and investor, and government. Some products are exported, and some are used as transport margins, which are added to the basic price of products. The total costs of domestic outputs (the sum of the first column) is equal to the total sales of domestic products (the sum of the first row).

As the owner of primary factors, the household receives factor incomes. Household primary income is divided between consumption expenditure and savings. Government collects various taxes and uses this income to pay for expenditures on goods and services.

Investment is partly funded by domestic savings. The gap between investment and saving is net foreign investment (NFI) inflow (18,191), which is equal to the trade deficit (316,949 – 298,758).

The data on household income and expenditure are extracted from the 2009-10 HES and SIH (ABS, 2011). The HES-SIH data are contained in two unit record data files: one file contains expenditure data on 9,774 sample households; the other file contains data on income sources for the 17,919 individual members of the sample households. The household data are mapped to the income sources and commodity categories of the CGE model.

On the income side, HES-SIH income items are aggregated to 33 types of personal income items: wage incomes from 8 labour occupations, capital income, 23 government benefits, and income taxes. In households with multiple income earners, incomes from each source are consolidated within each household. Table B.2 displays the allocation of the HES-SIH sources of income to the CGE database items.

Table B.2: Sources of household income in CGE model and HES-SIH.

CGE model	Household Expenditure Survey (HES-SIH)	
Wages for eight occupations (same as those in HES-SIH)	Managers and administrators; Professionals; Technicians and trade workers; Community and personal service workers;	Clerical and administrative workers; Sales workers; Machinery operators and drivers; Labourers;
Capital	Own unincorporated business; Investment;	Superannuation/Annuity/Private pension and other regular sources
Transfer payments from government	Austudy/Abstudy Age pension Carer allowance Carer payment Disability pension Disability support pension Family tax benefits Newstart allowance Other pensions and allowances Overseas pensions and benefits Parenting payment Baby bonus payment;	Partner allowance Service pension Sickness allowance Special benefit War widows pension Widow allowance Wife pension Youth allowance Utilities allowance Senior supplement Pension supplement
Direct taxes	Income taxes	

Source: Australian Bureau of Statistics, 2012.

On the expenditure side, there are 614 items recorded under the Commodity Code (ComCode) system in the HES data file. Among them, 485 can be aggregated into corresponding commodities in the input-output table. The rest need to be allocated to more than one commodity in the input-output table. In the absence of additional information about the composition of these commodities, the value shares of the commodities in the input-output table are used to allocate expenditures on these items.

Once the preliminary processing of the HES data is complete, a five-step procedure reconciles the differences between the two databases (Zhang, 2015). As the HES expenditure and income data are expressed in purchaser prices, they are aligned with input-output data in purchaser prices. The five steps are:

1. Use the HES weights to inflate the sample household data to the value for the whole population.
2. Scale the aggregated HES expenditure to be consistent with the total household expenditure in the input-output table.
3. Use the same scaling factor to scale the aggregated HES primary incomes.
4. Adjust each HES household's expenditure shares, while keeping total expenditure on each product constant, so that the aggregation of all HES expenditures is consistent with the commodity consumption pattern of the representative household shown in the input-output table.
5. In each industry in the input-output table, adjust factor income shares (labour by occupation and capital), while keeping the total value of factor income constant, so that the aggregated labour and capital income (net of capital savings) in the input-output table are consistent with the aggregated labour and capital incomes in the household data.

The aim of the second and third steps is to preserve the original relativities between households in the survey data. The aim of the fourth and fifth steps is to keep the budget balances of individual households and the factor returns of individual industries in the input-output table unchanged. A bi-proportional adjustment technique, such as RAS, or an entropy-based method, is required to carry out the last two steps.

A comparison between factor income in the 2012-13 input-output table and household income in the 2009-10 HES-SIH data is shown in Table B.3. A scaling factor of 1.3634 is used to update the 2009-10 HES-SIH data to the level consistent with the 2012-13 input-output table.

Table B.3: Comparison between CGE and HES-SIH aggregates (AUD million).

	CGE data (1)	HES-SIH scaled (2)	HES-SIH original
Expenditure	830,924	830,924	609,468
Income	1,371,109	850,486	623,816
Labour income	744,282	744,282	545,918
- Managers & administrators	132,202	132,202	96,968
- Professionals	229,510	229,510	168,341
- Technicians & trade workers	101,273	101,273	74,282
- Community & personal service	45,706	45,706	33,525
- Clerical & administrative workers	91,417	91,417	67,052
- Sales workers	43,234	43,234	31,712
- Machinery operators & drivers	50,821	50,821	37,277
- Labourers	50,118	50,118	36,761
Capital income	626,826	154,400	113,250
Benefits (23)		106,875	78,391
Income Tax (-)		155,071	113,742
Savings	540,185	19,562	14,348

Notes: 1. Capital income in the input-output table is divided between household (154,400) and firm saving (472,426). 2. Scaling factor between columns (2) and (3) = 1.3634.

C THE MODEL STRUCTURE

The core equations of the model are provided in this section, which is followed by a brief discussion of the model's theoretical structure. The sets used in the model equations are listed as follows.

Demands for domestic and imported goods (Equations 1-10)

Domestic price of imported good c, exclusive of duty

$$P_{(c,"imp")} = P^{cif}_{(c)}\, ex \qquad (c \in \mathrm{COM}) \tag{1}$$

where ex is the nominal exchange rate (domestic currency value per unit of foreign currency), and $P^{cif}_{(c)}$ is the world price of import c.

Basic price of good c from source s for user u

$$P^{b}_{(c,u,s)} = \begin{cases} P_{(c,"dom")} & (c \in \mathrm{COM};\ u \in \mathrm{USR};\ s = dom) \\ P_{(c,"imp")}\,(1 + t^{imp}_{(c,u)}) & (c \in \mathrm{COM};\ u \in \mathrm{USR};\ s = imp) \end{cases} \tag{2}$$

where $t^{imp}_{(c,u)}$ is the *ad valorem* rate of duty on imported good c for user u, and $P_{(c,"dom")}$ is the basic price of domestic good c.

CES demand of user u for good c from source s (exclusive of *stk*)

$$Q_{(c,u,s)} = CES(Q_{\_s(c,u)}, P^{b}_{(c,u,s)}, P^{b}_{\_s(c,u)}) \qquad (c\in\text{COM};\ u\in\text{USR\_stk};\ s\in\text{SRC}) \tag{3}$$

CES price for composite good c for user u

$$P^{b}_{\_s(c,u)} = \begin{cases} CES(P^{b}_{(c,u,"dom")}, P^{b}_{(c,u,"imp")}) & (c\in\text{COM};\ u\in\text{USR\_stk}) \\ \dfrac{1}{\sum_{s} Q_{(c,u,s)}} \sum_{s} Q_{(c,u,s)}\, P^{b}_{(c,u,s)} & (c\in\text{COM};\ u{=}\text{stk}) \end{cases} \tag{4}$$

Demand of good c for user u for margin good m

$$Q^{mgn}_{(c,u,m)} = Leontief(Q_{\_s(c,u)}) \qquad (c\in\text{COM};\ u\in\text{USR};\ m\in\text{MCM}) \tag{5}$$

Producers' price of composite good c for user u

$$P^{t\_}_{(c,u)} = \frac{1}{Q_{\_s(c,u)} + \sum_{m} Q^{mgn}_{(c,u,m)}} \left(Q_{\_s(c,u)}\, P^{b}_{\_s(c,u)} + \sum_{m} Q^{mgn}_{(c,u,m)}\, P_{(m,"dom")}\right) \quad (c\in\text{COM};\ u\in\text{USR}) \tag{6}$$

Purchasers' price of composite good c for user u

$$P^{t}_{(c,u)} = P^{t\_}_{(c,u)} \left(1 + \sum_{g} t^{dom}_{(c,u,g)}\right) \qquad (c\in\text{COM};\ u\in\text{USR}) \tag{7}$$

where $t^{dom}_{(c,u,g)}$ is the *ad valorem* rate of domestic tax g on composite good c for user u.

Producers' price of exported good c

$$P^{exp}_{(c)} = \frac{1}{Q^{exp}_{(c)} + \sum_{m} Q^{emgn}_{(c,m)}} \left(Q^{exp}_{(c)}\, P_{(c,"dom")} + \sum_{m} Q^{emgn}_{(c,m)}\, P_{(m,"dom")}\right) \qquad (c\in\text{COM}) \tag{8}$$

Foreign demand for exported good c

$$Q^{exp}_{(c)} = E^{exp}_{(c)} \left(P^{fob}_{(c)}\right)^{-\varepsilon_{(c)}} \qquad (c\in\text{COM}) \tag{9}$$

where $E^{exp}_{(c)}$ is foreign expenditure on export c, denoted in foreign currency, and $\varepsilon_{(c)}$ is the constant elasticity of demand for exported good c.

Foreign price of exported good *c*

$$P^{fob}_{(c)} = \frac{1}{ex} P^{exp}_{(c)} (1 + \sum_{g} t^{exp}_{(c,g)}) \qquad (c \in \mathrm{COM}) \tag{10}$$

where $t^{exp}_{(c,g)}$ is the *ad valorem* rate of tax *g* on exported good *c*.

Final user incomes and demands for composite goods (Equations 11-17)

Household factor income

$$Y^{fac} = \sum_{o} P^{lab}_{(o)} X^{lab}_{(o)} + P^{cap} X^{cap} \tag{11}$$

where s^{cap} is the share of capital income firms retain for investment.

Household disposable income

$$Y^{hou} = (Y^{fac} + P^{t}_{\_c("hou")} X^{ben}) (1 - t^{hou}) \tag{12}$$

Expenditure income of final user *u*

$$E_{(u)} = \begin{cases} Y^{hou} (1 - s^{hou}_{\_s}) & (u=hou) \\ (Tax\ Revenue) (1 - s^{gov}) & (u=gov) \\ Y^{hou} s^{hou}_{("inv")} + (Tax\ Revenue)\, s^{gov} + \dfrac{P^{cap} X^{cap} s^{cap}}{1 - s^{cap}} + Y^{fdi} & (u=inv) \\ Y^{hou} s^{hou}_{("stk")} & (u=stk) \end{cases} \tag{13}$$

where s^{gov} is government saving rate and $s^{hou}_{\_s}$ is household saving rate,

$$s^{hou}_{\_s} = s^{hou}_{("inv")} + s^{hou}_{("stk")}$$

where $s^{hou}_{("inv")}$ and $s^{hou}_{("stk")}$ are household savings for investment and change in inventory, respectively.

Net foreign direct investment (in domestic currency)

$$Y^{fdi} = P^{t}_{\_c("inv")} Q^{fdi} \tag{14}$$

where Q^{fdi} is the real inflow of net foreign direct investment.

Purchasers' price index for user *u*

$$P^{t}_{\_c(u)} = \frac{1}{\sum_{c} Q_{\_s(c,u)}} \sum_{c} Q_{\_s(c,u)}\, P^{t}_{(c,u)} \qquad (u \in \mathrm{USR}) \tag{15}$$

Demand of user u for composite good c

$$Q_{\_s(c,u)} = \begin{cases} Leontief(X_{(u)}) & (c \in \mathrm{COM};\ u \in \mathrm{IND}) \\ Q^{hcom}_{\_h(c)} & (c \in \mathrm{COM};\ u = hou) \\ Leontief(\frac{E_{(u)}}{P^{t}_{\_c(u)}}) & (c \in \mathrm{COM};\ u = inv) \\ \sum_{s} Q_{(c,u,s)} & (c \in \mathrm{COM};\ u = stk) \end{cases} \tag{16}$$

where $X_{(u)}$ is the output of industry u and $Q_{\_s(c,"gov")}$ and $Q_{(c,"stk",s)}$ are exogenous variables.

Budget constraint for government and inventory $(=> s^{gov}, s^{hou}_{("stk")})$

$$E_{(u)} = \sum_{c} (P^{t}_{(c,u)}\, Q_{\_s(c,u)}) \qquad (u = gov,\ stk) \tag{17}$$

Industry outputs, prices and demands for factors (Equations 18-25)

CES demand of industry i for labour occupation o

$$Q^{lab}_{(o,i)} = CES(Q^{lab}_{\_o(i)}, P^{lab}_{(o)}, P^{fac}_{("lab",i)}) \qquad (o \in \mathrm{OCC};\ i \in \mathrm{IND}) \tag{18}$$

where $Q^{lab}_{\_o(i)}$ is the demand of industry i for composite labour,

$$Q^{lab}_{\_o(i)} = Leontief(Q^{fac}_{("lab",i)}) \qquad (i \in \mathrm{IND})$$

Basic price of factor f in industry i

$$P^{fac}_{(f,i)} = \begin{cases} CES(P^{lab}_{(o_1)}, \ldots, P^{lab}_{(o_8)}) & (f = lab;\ i \in \mathrm{IND}) \\ P^{cap} & (f = cap;\ i \in \mathrm{IND}) \end{cases} \tag{19}$$

where $P^{fac}_{("lab",i)}$ is a CES price index for composite labour, $P^{lab}_{(o)}$ and P^{cap} are the equilibrium prices of labour occupation o and capital, respectively.

CES demand of industry i for factor f

$$Q^{fac}_{(f,i)} = CES(Q^{fac}_{\_f(i)}, P^{fac}_{(f,i)}, P^{fac}_{\_f(i)}) \qquad (f \in \mathrm{FAC};\ i \in \mathrm{IND}) \tag{20}$$

where $Q^{fac}_{\_f(i)}$ is the demand of industry i for composite factor

$$Q^{fac}_{\_f(i)} = Leontief(X_{(i)}) \qquad (i \in \text{IND}) \tag{21}$$

CES price index for composite factor in industry *i*

$$P^{fac}_{\_f(i)} = CES(P^{fac}_{("cap",i)}, P^{fac}_{("lab",i)}) \qquad (i \in \text{IND}) \tag{22}$$

Unit cost of output in industry *i*

$$C^{ind}_{(i)} = \frac{1}{\sum_c Q_{\_s(c,i)} + \sum_f Q^{fac}_{(f,i)}} \sum_c Q_{\_s(c,i)} P^{t}_{(c,i)} + \sum_f Q^{fac}_{(f,i)} P^{fac}_{(f,i)} \qquad (i \in \text{IND}) \tag{22}$$

Basic price of output in industry *i*

$$P^{ind}_{(i)} = C^{ind}_{(i)} (1 + t^{ind}_{(i)}) \qquad (i \in \text{IND}) \tag{23}$$

where $t^{ind}_{(i)}$ is the *ad valorem* rate of a tax on output of industry *i*.

Total supply of labour occupation *o* (equal to its demand in equilibrium) $(=> P^{lab}_{(o)})$

$$X^{lab}_{(o)} = \sum_i Q^{lab}_{(o,i)} \qquad (o \in \text{OCC}) \tag{24}$$

Total supply of capital (equal to its demand in equilibrium) $(=> P^{cap})$

$$\frac{X^{cap}}{1 - s^{cap}} = \sum_i Q^{fac}_{("cap",i)} \tag{25}$$

Industry supplies of multi-products (Equations 26-29)

CET supply of good *c* from industry *i*

$$X^{com}_{(c,i)} = CET(X_{(i)}, P_{(c,"dom")}, P^{ind}_{(i)}) \qquad (c \in \text{COM};\ i \in \text{IND}) \tag{26}$$

CET price index for composite goods (equal to $P^{ind}_{(i)}$ in equilibrium) $(=> X_{(i)})$

$$P^{ind}_{(i)} = CET(P_{(c_1,"dom")}, \ldots, P_{(c_m,"dom")}) \qquad (i \in \text{IND}) \tag{27}$$

Total supply of good *c* from all industries

$$X^{com}_{\_i(c)} = \sum_i X^{com}_{(c,i)} \qquad (c \in \text{COM}) \tag{28}$$

Total sales of good c (equal to $X^{com}_{\_i(c)}$ in equilibrium) (=> $P_{(c,"dom")}$)

$$X^{com}_{\_i(c)} = \begin{cases} \sum_u Q_{(c,u,"dom")} + Q^{exp}_{(c)} & (c \in \text{NCM}) \\ \sum_u Q_{(c,u,"dom")} + Q^{exp}_{(c)} + \sum_n \sum_u Q^{mgn}_{(n,u,c)} + \sum_n Q^{emgn}_{(n,c)} & (c \in \text{MCM}) \end{cases} \quad (29)$$

Household income and expenditure (Equations 30-35)

Factor income from factor f for person n in household h

$$Y^{hfac}_{(f,h,n)} = \begin{cases} \sum_o P^{lab}_{(o)} X^{hlab}_{(o,h,n)} & (f=lab;\ h \in \text{HOU};\ n \in \text{PER}) \\ P^{cap} X^{hcap}_{(h,n)} & (f=cap;\ h \in \text{HOU};\ n \in \text{PER}) \end{cases} \quad (30)$$

where $X^{hlab}_{(o,h,n)}$ is labour supply from occupation o and $X^{hcap}_{(h,n)}$ is capital supply.

Taxable income for person n in household h

$$Y^{htinc}_{(h,n)} = \sum_f Y^{hfinc}_{(f,h,n)} + \sum_{b \in \text{B23}} P^{t}_{\_c("hou")} X^{hben}_{(b,h,n)} \qquad (h \in \text{HOU};\ n \in \text{PER}) \quad (31)$$

where $X^{hben}_{(b,h,n)}$ is the entitlement of real benefit b.

Disposable income for person n in household h

$$Y^{hdinc}_{(h,n)} = Y^{htinc}_{(h,n)} (1 - t^{hou}_{(h,n)}) \qquad (h \in \text{HOU};\ n \in \text{PER}) \quad (32)$$

where $t^{hou}_{(h,p)}$ is the rate of a tax on the taxable income of person n in household h.

Expenditure income for household h

$$Y^{heinc}_{(h)} = \sum_n Y^{hdinc}_{(h,n)} (1 - r^{hsav}_{(h)}) \qquad (h \in \text{HOU}) \quad (33)$$

Changes in saving rate for household h

$$\frac{d(r^{hsav}_{(h)})}{r^{hsav}_{(h)}} = \frac{d(s^{hou}_{\_s})}{s^{hou}_{\_s}} \qquad (h \in \text{HOU}) \quad (34)$$

Demand of household h for composite good c

$$Q^{hcom}_{(c,h)} = f(Y^{heinc}_{(h)}, P^{t2}_{\_s(c,"hou")}) \qquad (c \in \text{COM};\ h \in \text{HOU}) \quad (35)$$

Linking household variables with CGE aggregates (Equations 36-40)

Aggregation of household supplies of capital stock

$$X^{cap} = \sum_{h} \sum_{n} X^{hcap}_{(h,n)} \tag{36}$$

Aggregation of household supplies of labour occupation o

$$X^{lab}_{(o)} = \sum_{h} \sum_{n} X^{hlab}_{(o,h,n)} \qquad (o \in \mathrm{OCC}) \tag{37}$$

Aggregation of household demands for composite good c

$$Q^{hcom}_{\_h(c)} = \sum_{h} Q^{hcom}_{(c,h)} \qquad (c \in \mathrm{COM}) \tag{38}$$

Average household income tax rate

$$t^{hou} = \frac{\sum_{h} \sum_{n} Y^{htinc}_{(h,n)} \, t^{hou}_{(h,n)}}{\sum_{h} \sum_{n} Y^{htinc}_{(h,n)}} \tag{39}$$

Aggregation of household real benefits

$$X^{ben} = \sum_{b \in \mathrm{B23}} \sum_{h} \sum_{n} X^{hben}_{(b,h,n)} \tag{40}$$

Table C.1: Sets used in the model and database.

Sets	Definitions
COM(1,…,*m*):	Commodities (indexed by *c*)
IND(1,…,*m*):	Industries (indexed by *i*)
MCM(1,…,*n*):	Margin commodities (indexed by *m*)
NCM(= COM − MCM):	Non-margin commodities (indexed by *c*)
FUSR(*gov*, *hou*, *inv*, *stk*):	Final users of commodities (indexed by *u*)
USR(=IND + FUSR):	Users of commodities (indexed by *u*)
USR_stk(=USR − *stk*):	Users of commodities, exclusive of *stk* (indexed by *u*)
SRC(*dom*, *imp*):	Domestic/import sources of commodities (indexed by *s*)
FAC(*lab*,*cap*):	Factors of production (indexed by *f*)
OCC(1,…,8):	Labour occupation (indexed by *o*)
SAV(*inv*,*stk*):	Saving destination (indexed by *s*)
HOU(1,…,9774):	Sample households (indexed by *h*)
PER(1,…,6):	Persons in a household (indexed by *p*)
B23(1,…,23):	Transfer payments (indexed by *b*)
TAX(*GST*, *tax*, *sub*):	Three tax items (indexed by *g*)

The model specifies the behaviours of three types of economic agents:

- A representative firm in each industry, which purchases intermediate inputs and factor services to produce goods or services. For given demand for its products and input prices, the firm uses constant-return-to-scale (CRTS) technology to minimise the costs of production.
- A household sector, which plays three roles: as factor owner, it receives factor incomes from firms; as consumer, it spends income on goods and services to maximise its utility; and as investor, it allocates savings to form new capital.
- A government sector, which collects revenue from taxes on goods and services and from household and spends income on transfers to households and on purchasing goods and services.

The price system consists of three layers:

- Basic price = unit cost of production, including intermediate and factor input costs, plus production tax; the basic price of imported goods include import tariffs.
- Producer price = basic price plus transport and sales margin services.
- Purchaser price = producer price plus indirect taxes.

The basic structure of the model can be summarised in the behaviours of two groups of users: users of intermediate inputs (firms) and users of final products (household, government and investor).

Firm behaviour has five components (refer to Table B.1).

- *Industry demand for domestic and imported intermediate inputs*: a CES cost-minimising demand function of two variables – the given purchasers' prices and the industry demand for the composite good (Equation 3).
- *Industry demand for composite intermediate inputs*: a Leontief cost-minimising demand function of the industry's firms (Equation 16).
- *Industry demand for factor inputs*: a CES cost-minimising demand function of two variables – basic factor prices and the industry's output (Equations 18-21).

The input-output table used for this model is a *product-industry* table, where some industries produce multiple products. To model this production structure, a CET supply of multiple products is introduced in each industry (Equation 26), which is a function of the industry's total output, $X_{(i)}$, and the basic prices of the products produced, $P_{(c,"dom")}$. The general equilibrium output of each industry $X_{(i)}$ is determined by linking the associated CET price index with the basic price of the industry $P_{(i)}^{ind}$ in Equation 27. The general equilibrium basic price of each product, $P_{(c,"dom")}$, is determined in Equation 29 by its market clearing condition.

Final users' behaviours (household, government and investor) have three components.

- *Final user's demand for domestic and imported goods* (Equation 3): a CES cost-minimising demand function of two variables – the given purchasers' prices and final user's demand for the composite good.

- *Final user's demand for composite goods* (Equation 16): the functional form varies depending on the final user concerned. For investors, it is a Leontief cost-minimising demand function of capital formation. Government demand is assumed to be exogenously given. For households, it is an aggregation of individual household demands to be defined in the household module below.
- *Final users' income and expenditure* (Equation 13): as indicated earlier in the model data section, the three final users all have their expenditures fully paid for by their respective incomes, and, therefore, all have balanced budgets.

As government and inventory demands are exogenously given, two budget constraints are required, so that government saving rate s^{gov} and household saving rate for inventory $s^{hou}_{("stk")}$ are set as endogenous to balance their respective budgets (Equation 17).

The foreign demand for exported goods is defined in a constant elasticity of demand function (Equation 9).

There are two components in the household module, specified in Equations 30 to 35. On the income side, each person's incomes from various sources are aggregated within a household. Four types of income are defined.

- *Factor income*, composed of labour income from different occupation categories and from capital (Equation 30).
- *Taxable income*, composed of factor income plus various benefits received from the government and transfer income from other households, as derived from the HES-SIH datasets (Equation 31).
- *Disposable income*, defined as taxable income, net of income tax (Equation 32).
- *Expenditure income*, referring to the part of household disposable income that is used for current consumption (Equation 33). The remainder is household saving, which is assumed to be adjusted by changes in the average saving rate $s^{hou}_{\_s}$ (Equation 34).

On the expenditure side, household demands for the composites of domestically produced and imported goods are derived from a Cobb-Douglas utility function, which is a function of household income and goods purchaser prices (Equation 35). As household incomes and

consumption patterns (which determine the parameters in the demand functions) are different, each household has a unique and individual response to a given set of price changes. This functional form is chosen for simplicity consideration. Other more flexible functional forms may also be used, but are unlikely to alter the model's convergence behaviour.

Household factor endowments can be aggregated and linked to the variables of factor supplies in the main model (Equations 36-40). The household supplies of factors are assumed to be fixed in the integrated model. This assumption is relaxed when using a separate BMS model in Section 3 of this paper.

In this model, the following variables are set as exogenous

- All tax rates and real benefits: $t^{imp}_{(c,u)}$, $t^{exp}_{(c,g)}$, $t^{ind}_{(i)}$, $t^{dom}_{(c,u,g)}$, $t^{hou}_{(h,n)}$ and $X^{hben}_{(b,h,n)}$
- Household and firm saving rates: $s^{hou}_{("inv")}$ and s^{cap}
- Household supplies of capital and labour: $X^{hcap}_{(h,n)}$ and $X^{hlab}_{(o,h,n)}$
- The world prices of imports, foreign income and net foreign investment: $P^{cif}_{(i)}$, $E^{exp}_{(i)}$ and Q^{fdi}
- Foreign exchange rate *ex*, which is also used as the numeraire

Other variables are all endogenously defined by an equation, except the following three groups of variables, which are not defined by any equation.

- Government saving rate s^{gov} and household saving rate for inventory $s^{hou}_{("stk")}$
- Industry output $X_{(i)}$ and product basic price $P_{(c,"dom")}$
- Basic prices for labour (by occupation) $P^{lab}_{(o)}$ and capital P^{cap}

The equilibrium values of first two groups of variables are determined by Equations 17, 27 and 29, respectively, which have been discussed above, not to be repeated here. The basic prices of labour and capital are determined by their respective market clearing conditions given in Equations 24 and 25.

As a result, the model has equal numbers of endogenous variables and equations, implying a valid closure and a unique solution. A test simulation confirms that this is the case.